Spiral cylindrique avec courbes terminales : 2 arcs de cercle et une droite

Développement excentrique et anisochronisme en position horizontale

Déformations planes

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm \quad TOL := 10^{-12}$

Elinvar $\quad \rho_s = 8\times 10^3\ kg\cdot m^{-3} \quad E = 1.7\times 10^{11}\,Pa \quad G = 6.538\times 10^{10}\,Pa$

Partie cylindrique $\quad n_s := 10.15 \quad \psi_0 := n_s\cdot 360\cdot deg \quad \psi_0 = 3.654\times 10^3\,deg \quad L := R_0\cdot\psi_0 \quad L = 318.872\,mm$

$r_s(\alpha) := R_0 \quad s(\alpha) := R_0\cdot(\alpha-\pi) \quad x_{0s}(\alpha) := R_0\cdot cos(\alpha) \quad y_{0s}(\alpha) := R_0\cdot sin(\alpha)$

Courbe terminale externe $\quad r_t := 0.5\cdot R_0 \quad l_t := R_0 + \pi\cdot r_t \quad \alpha_A := \pi$

$x_{0t1}(\alpha_t) := r_t\cdot(1+cos(\alpha_t)) \quad y_{0t1}(\alpha_t) := r_t\cdot sin(\alpha_t) \quad x_{0t2}(x) := x \quad y_{0t2}(x) := r_t$

$x_{0t3}(\beta_t) := -r_t\cdot(1+sin(\beta_t)) \quad y_{0t3}(\beta_t) := r_t\cdot cos(\beta_t)$

Courbe terminale interne $\quad \alpha_B := mod(\psi_0+\pi, 2\cdot\pi) \quad \alpha_B = 234\,deg$

$x_{0t'1}(\alpha_{t'}) := x_{0t1}(\alpha_{t'})\cdot cos(\alpha_B) - y_{0t1}(\alpha_{t'})\cdot sin(\alpha_B) \quad x_{0t'2}(x) := x_{0t2}(x)\cdot cos(\alpha_B) - y_{0t2}(x)\cdot sin(\alpha_B)$

$y_{0t'1}(\alpha_{t'}) := x_{0t1}(\alpha_{t'})\cdot sin(\alpha_B) + y_{0t1}(\alpha_{t'})\cdot cos(\alpha_B) \quad y_{0t'2}(x) := x_{0t2}(x)\cdot sin(\alpha_B) + y_{0t2}(x)\cdot cos(\alpha_B)$

$x_{0t'3}(\beta_{t'}) := x_{0t3}(\beta_{t'})\cdot cos(\alpha_B) - y_{0t3}(\beta_{t'})\cdot sin(\alpha_B)$

$y_{0t'3}(\beta_{t'}) := x_{0t3}(\beta_{t'})\cdot sin(\alpha_B) + y_{0t3}(\beta_{t'})\cdot cos(\alpha_B) \quad L_t := 2\cdot l_t + L$

Position du piton $\quad r_P := R_0 \quad \alpha_P := 0 \quad x_P := R_0 \quad y_P := 0\cdot mm$

Position de la virole $\quad r_V := R_0 \quad \alpha_V(\theta) := mod(\alpha_B+\pi+\theta, 2\cdot\pi) \quad \alpha_V(0) = 54\,deg$

$x_V(\theta) := r_V\cdot cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V\cdot sin(\alpha_V(\theta))$

Amplitude stationnaire du balancier $\quad \theta_0 := 270\cdot deg$

Contrainte maximum

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha) \quad W_{f3} := W_{f\_rect}(ép, ha) \quad \sigma_{max} := \frac{E\cdot I_{33}}{L\cdot W_{f3}}\cdot\theta_0 \quad \sigma_{max} = 113.054\,\frac{N}{mm^2}$

Centres de masse

Partie cylindrique $\quad z_{0s}(\alpha) := x_{0s}(\alpha) + i\cdot y_{0s}(\alpha)$

$\zeta_{0s} := \frac{R_0}{L}\cdot\int_{\pi}^{\psi_0+\pi} z_{0s}(\alpha)\,d\alpha \quad \xi_{0s} := Re(\zeta_{0s}) \quad \eta_{0s} := Im(\zeta_{0s}) \quad \xi_{0s} = -0.063\,mm \quad \eta_{0s} = -0.032\,mm$

Courbe terminale externe

$z_{0t1}(\alpha_t) := x_{0t1}(\alpha_t) + i\cdot y_{0t1}(\alpha_t) \quad z_{0t2}(x) := x_{0t2}(x) + i\cdot y_{0t2}(x) \quad z_{0t3}(\beta_t) := x_{0t3}(\beta_t) + i\cdot y_{0t3}(\beta_t)$

$$\zeta_{0t} := \frac{1}{l_t}\cdot\left(\int_0^{\frac{\pi}{2}} z_{0t1}(\alpha_t)\cdot r_t\,d\alpha_t - \int_{r_t}^{-r_t} z_{0t2}(x)\,dx + \int_0^{\frac{\pi}{2}} z_{0t3}(\beta_t)\cdot r_t\,d\beta_t\right)$$

$\xi_{0t} := Re(\zeta_{0t})$ $\quad\eta_{0t} := Im(\zeta_{0t})$ $\quad\xi_{0t} = 0\,mm$ $\quad\eta_{0t} = 1.945\,mm$

Courbe terminale interne

$z_{0t'1}(\alpha_{t'}) := x_{0t'1}(\alpha_{t'}) + i\cdot y_{0t'1}(\alpha_{t'})$ $\quad z_{0t'2}(x) := x_{0t'2}(x) + i\cdot y_{0t'2}(x)$ $\quad z_{0t'3}(\beta_{t'}) := x_{0t'3}(\beta_{t'}) + i\cdot y_{0t'3}(\beta_{t'})$

$$\zeta_{0t'} := \frac{1}{l_t}\cdot\left(\int_0^{\frac{\pi}{2}} z_{0t'1}(\alpha_{t'})\cdot r_t\,d\alpha_{t'} - \int_{r_t}^{-r_t} z_{0t'2}(x)\,dx + \int_0^{\frac{\pi}{2}} z_{0t'3}(\beta_{t'})\cdot r_t\,d\beta_{t'}\right)$$

$\xi_{0t'} := Re(\zeta_{0t'})$ $\quad\eta_{0t'} := Im(\zeta_{0t'})$ $\quad\xi_{0t'} = 1.573\,mm$ $\quad\eta_{0t'} = -1.143\,mm$

Centre de masse du spiral $\quad\zeta_s := \frac{1}{L_t}\cdot(L\cdot\zeta_{0s} + l_t\cdot\zeta_{0t} + l_t\cdot\zeta_{0t'})$ $\quad\zeta_s = 0\,mm$

Première approximation de la déformée du spiral

Courbe terminale externe

$\varphi_{0t1}(\alpha_t) := \alpha_t + \frac{\pi}{2}$ $\quad z_P := x_P + i\cdot y_P$ $\quad z_{1t1}(\theta,\alpha_t) := z_P + r_t\cdot\int_0^{\alpha_t} i\cdot e^{i\cdot\alpha'_t}\cdot exp\left(i\cdot\theta\cdot\frac{r_t}{L_t}\cdot\alpha'_t\right)d\alpha'_t$

$$z_{1t1}(\theta,\alpha_t) := z_P + \frac{r_t\cdot L_t}{L_t + \theta\cdot r_t}\cdot\left(exp\left(i\cdot\frac{\alpha_t\cdot L_t + \theta\cdot r_t\cdot\alpha_t}{L_t}\right) - 1\right)$$

$\varphi_{0t2} := \pi$ $\quad \Delta z_{1t2}(\theta,x) := \int_{r_t}^{x} exp\left[i\cdot\frac{\theta}{L_t}\cdot(r_t - x')\right]dx'$

$\Delta z_{1t2}(\theta,x) := i\cdot\frac{L_t}{\theta}\cdot\left(exp\left(-i\cdot\frac{x - r_t}{L_t}\cdot\theta\right) - 1\right)$ $\quad z_{1C}(\theta) := z_{1t1}\left(\theta,\frac{\pi}{2}\right)$

$\Delta\varphi_{1C}(\theta) := \frac{\theta}{L_t}\cdot\frac{\pi\cdot r_t}{2}$ $\quad\Delta\varphi_{1C}(\theta_0) = 3.077\,deg$ $\quad z_{1t2}(\theta,x) := z_{1C}(\theta) + \Delta z_{1t2}(\theta,x)\cdot e^{i\cdot\Delta\varphi_{1C}(\theta)}$

$\varphi_{0t3}(\beta_t) := \beta_t + \frac{\pi}{2}$ $\quad\Delta z_{1t3}(\theta,\beta_t) := r_t\cdot\int_0^{\beta_t} i\cdot e^{i\cdot\beta'_t}\cdot exp\left[i\cdot\frac{\theta}{L_t}\cdot(r_t\cdot\beta'_t)\right]d\beta'_t$

$\Delta z_{1t3}(\theta,\beta_t) := \frac{r_t\cdot L_t}{L_t + \theta\cdot r_t}\cdot\left(exp\left(i\cdot\beta_t\cdot\frac{L_t + \theta\cdot r_t}{L_t}\right) - 1\right)$ $\quad z_{1D}(\theta) := z_{1t2}(\theta,-r_t)$

$\Delta\varphi_{1D}(\theta) := \frac{\theta}{L_t}\cdot\left(\frac{\pi\cdot r_t}{2} + R_0\right)$ $\quad\Delta\varphi_{1D}(\theta_0) = 6.995\,deg$ $\quad z_{1t3}(\theta,\beta_t) := z_{1D}(\theta) + \Delta z_{1t3}(\theta,\beta_t)\cdot e^{i\cdot\left(\Delta\varphi_{1D}(\theta)+\frac{\pi}{2}\right)}$

Partie cylindrique

$$\varphi_0(\alpha') := \alpha' + \frac{\pi}{2} \qquad \Delta z_{1s}(\theta,\alpha) := R_0 \cdot \int_{\pi}^{\alpha} i \cdot exp(i \cdot \alpha') \cdot exp\left(i \cdot \theta \cdot R_0 \cdot \frac{\alpha' - \pi}{L_t}\right) d\alpha'$$

$$\Delta z_{1s}(\theta,\alpha) := \frac{R_0 \cdot L_t}{L_t + \theta \cdot R_0} \cdot \left(exp\left(-i \cdot \frac{-\alpha \cdot L_t - \theta \cdot R_0 \cdot \alpha + \theta \cdot R_0 \cdot \pi}{L_t}\right) + 1\right) \qquad z_{1A}(\theta) := z_{1t3}\left(\theta, \frac{\pi}{2}\right)$$

$$\Delta\varphi_{1A}(\theta) := \theta \cdot \frac{l_t}{L_t} \qquad \Delta\varphi_{1A}(\theta_0) = 10.072\, deg \qquad z_{1s}(\theta,\alpha) := z_{1A}(\theta) + \Delta z_{1s}(\theta,\alpha) \cdot e^{i \cdot \Delta\varphi_{1A}(\theta)}$$

Courbe terminale interne

$$\Delta z_{1t'1}(\theta,\alpha_{t'}) := r_{t'} \cdot \int_0^{\alpha_{t'}} i \cdot exp(i \cdot \alpha'_{t'}) \cdot exp\left(i \cdot \theta \cdot \frac{r_t}{L_t} \cdot \alpha'_{t'}\right) d\alpha'_{t'}$$

$$\Delta z_{1t'1}(\theta,\alpha_{t'}) := \frac{r_t \cdot L_t}{\theta \cdot r_t + L_t} \cdot \left(exp\left(i \cdot \alpha_{t'} \cdot \frac{\theta \cdot r_t + L_t}{L_t}\right) - 1\right) \qquad z_{1B}(\theta) := z_{1s}(\theta, \psi_0 + \pi) \qquad \alpha_B = 234\, deg$$

$$\alpha_{1B}(\theta) := Atan(Re(z_{1B}(\theta)), Im(z_{1B}(\theta))) \qquad z_{1t'1}(\theta,\alpha_{t'}) := z_{1B}(\theta) + \Delta z_{1t'1}(\theta,\alpha_{t'}) \cdot e^{i \cdot \alpha_{1B}(\theta)}$$

$$\Delta z_{1t'2}(\theta,x') := i \cdot \frac{L_t}{\theta} \cdot \left(exp\left(-i \cdot \frac{x' - r_t}{L_t} \cdot \theta\right) - 1\right) \qquad z_{1C'}(\theta) := z_{1t'1}\left(\theta, \frac{\pi}{2}\right)$$

$$\Delta\varphi_{1C'}(\theta) := \alpha_{1B}(\theta) + \Delta\varphi_{1C}(\theta) \qquad z_{1t'2}(\theta,x) := z_{1C'}(\theta) + \Delta z_{1t'2}(\theta,x) \cdot e^{i \cdot \Delta\varphi_{1C'}(\theta)}$$

$$\Delta z_{1t'3}(\theta,\beta_{t'}) := \frac{r_t \cdot L_t}{L_t + \theta \cdot r_t} \cdot \left(exp\left(i \cdot \beta_{t'} \cdot \frac{L_t + \theta \cdot r_t}{L_t}\right) - 1\right) \qquad z_{1D'}(\theta) := z_{1t'2}(\theta, -r_t)$$

$$\Delta\varphi_{1D'}(\theta) := \frac{\theta}{L_t} \cdot \left(\frac{\pi \cdot r_t}{2} + R_0\right) + \alpha_{1B}(\theta) \qquad z_{1t'3}(\theta,\beta_{t'}) := z_{1D'}(\theta) + \Delta z_{1t'3}(\theta,\beta_{t'}) \cdot e^{i \cdot \left(\Delta\varphi_{1D'}(\theta) + \frac{\pi}{2}\right)}$$

Graphe de la déformation

Forme naturelle

$$n_t := 201 \qquad j := 0 .. n_t - 1 \qquad \Delta\alpha_t := \frac{\pi}{2 \cdot (n_t - 1)} \qquad \alpha_{t_j} := j \cdot \Delta\alpha_t \qquad x_{t1_j} := x_{0t1}(\alpha_{t_j}) \qquad y_{t1_j} := y_{0t1}(\alpha_{t_j})$$

$$x_j := r_t - j \cdot \frac{2 \cdot r_t}{n_t - 1} \qquad x_{t2_j} := x_{0t2}(x_j) \qquad y_{t2_j} := y_{0t2}(x_j) \qquad x_0 := pile(x_{t1}, x_{t2}) \qquad y_0 := pile(y_{t1}, y_{t2})$$

$$\beta_{t_j} := j \cdot \Delta\alpha_t \qquad x_{t3_j} := x_{0t3}(\beta_{t_j}) \qquad y_{t3_j} := y_{0t3}(\beta_{t_j}) \qquad x_t := pile(x_0, x_{t3}) \qquad y_t := pile(y_0, y_{t3})$$

$$n := 20 \cdot partentière(n_s) + 1 \qquad i := 0 .. n - 1 \qquad \Delta\alpha := \frac{\psi_0}{n - 1} \qquad \alpha_i := \pi + i \cdot \Delta\alpha$$

$$x_{s_i} := x_{0s}(\alpha_i) \qquad y_{s_i} := y_{0s}(\alpha_i) \qquad x_0 := pile(x_t, x_s) \qquad y_0 := pile(y_t, y_s)$$

$$\alpha_{t'_j} := j \cdot \Delta\alpha_t \qquad x_{t'1_j} := x_{0t'1}(\alpha_{t'_j}) \qquad y_{t'1_j} := y_{0t'1}(\alpha_{t'_j}) \qquad x_0 := pile(x_0, x_{t'1}) \qquad y_0 := pile(y_0, y_{t'1})$$

$$x_{t'2_j} := x_{0t'2}(x_j) \qquad y_{t'2_j} := y_{0t'2}(x_j) \qquad x_0 := pile(x_0, x_{t'2}) \qquad y_0 := pile(y_0, y_{t'2})$$

$$\beta_{t'_j} := j \cdot \Delta\alpha_t \qquad x_{t'3_j} := x_{0t'3}(\beta_{t'_j}) \qquad y_{t'3_j} := y_{0t'3}(\beta_{t'_j}) \qquad x_0 := pile(x_0, x_{t'3}) \qquad y_0 := pile(y_0, y_{t'3})$$

$r_0 := \overrightarrow{\sqrt{{x_0}^2 + {y_0}^2}}$ $\qquad \beta_s := \overrightarrow{Atan(x_0, y_0)}$

Déformée

$z_{td1} := \overrightarrow{z_{1t1}(\theta_0, \alpha_t)}$ $\quad z_{td2_j} := z_{1t2}(\theta_0, x_j)$ $\quad z_d := pile(z_{td1}, z_{td2})$ $\quad z_{td3_j} := z_{1t3}(\theta_0, \beta_{t_j})$ $\quad z_d := pile(z_d, z_{td3})$

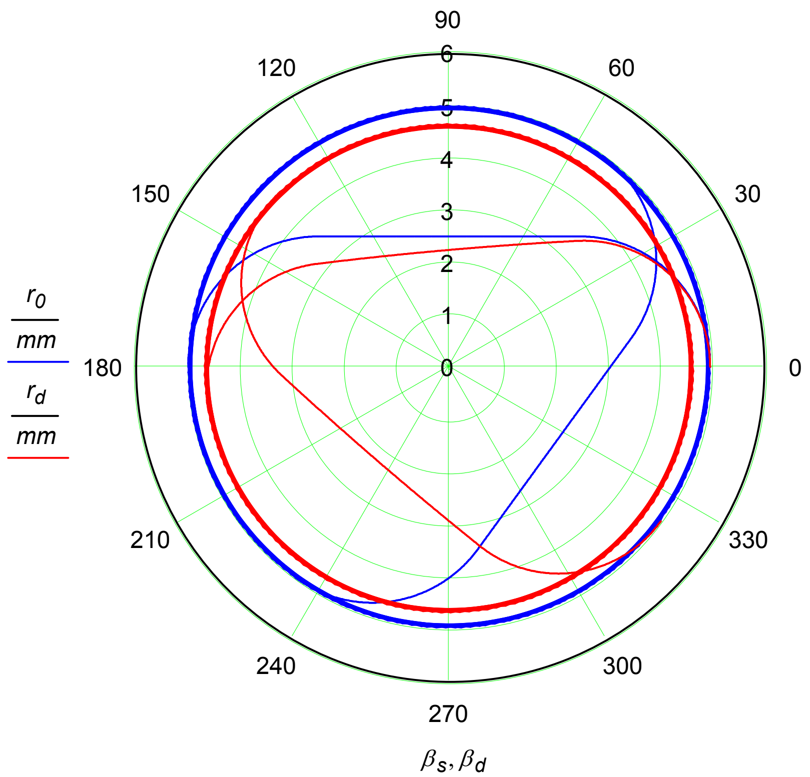
$z_{sd} := \overrightarrow{z_{1s}(\theta_0, \alpha)}$ $\quad z_d := pile(z_d, z_{sd})$

$z_{t'd1} := \overrightarrow{z_{1t'1}(\theta_0, \alpha_{t'})}$ $\quad z_d := pile(z_d, z_{t'd1})$ $\quad z_{t'd2} := \overrightarrow{z_{1t'2}(\theta_0, x)}$ $\quad z_d := pile(z_d, z_{t'd2})$

$z_{t'd3} := \overrightarrow{z_{1t'3}(\theta_0, \beta_{t'})}$ $\quad z_d := pile(z_d, z_{t'd3})$ $\quad n_{pt} := dernier(z_d)$

$x_d := Re(z_d)$ $\quad y_d := Im(z_d)$ $\quad r_d := \overrightarrow{|z_d|}$ $\quad r_{d_{npt}} = 5.022\,mm$

$\beta_d := \overrightarrow{Atan(x_d, y_d)}$ $\quad \beta_{d_0} = 0\,deg$ $\quad \beta_{d_{npt}} = 323.924\,deg$ $\quad mod(\alpha_V(\theta_0), 2\cdot\pi) = 324\,deg$



$mod(\psi_0, 2\cdot\pi) = 54\,deg$

$r_P = 5\,mm$

$r_V = 5\,mm$

$\alpha_V(0) = 54\,deg$

$x_V(\theta_0) = 4.045\,mm$

$y_V(\theta_0) = -2.939\,mm$

$\Delta x_V := x_{d_{npt}} - x_V(\theta_0)$

$\Delta x_V = 0.014\,mm$

$\Delta y_V := y_{d_{npt}} - y_V(\theta_0)$

$\Delta y_V = -0.018\,mm$

Déplacement de la virole libre

Contribution du spiral sans ses courbes terminales

$s_s(\alpha) := R_0\cdot(\alpha - \pi) + l_t$ $\qquad f_s(\theta, \alpha) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)$

$\mathbf{\Delta_S}(\theta) := \frac{R_0}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} z_{0s}(\alpha)\cdot f_s(\theta, \alpha)\, d\alpha$ $\qquad \mathbf{\Delta_S}(\theta_0) = 0.093 + 0.286i\,mm$

Approximation $\quad \mathbf{OA} := R_0\cdot e^{i\cdot\pi}$ $\quad \mathbf{OB} := R_0\cdot e^{i\cdot(\pi+\psi_0)}$ $\quad f'_s(\theta, \alpha) := \frac{-\theta^2}{L_t}\cdot R_0\cdot exp\left(i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)$

$$\Delta_{as}(\theta) := \frac{R_0}{L_t}\cdot\left[\left(i\cdot f_s(\theta,\pi) - f'_s(\theta,\pi)\right)\cdot \mathbf{OA} + \left(-i\cdot f_s(\theta,\pi+\psi_0) + f'_s(\theta,\pi+\psi_0)\right)\cdot \mathbf{OB}\right]$$

$$\Delta_{as}(\theta) := \frac{R_0}{L_t}\cdot\theta\cdot e^{i\cdot\theta\cdot\frac{l_t}{L_t}}\cdot\left(-\mathbf{OA} + e^{i\cdot\theta\cdot\frac{L}{L_t}}\cdot\mathbf{OB}\right) + \frac{R_0^{\,2}}{L_t^{\,2}}\cdot\theta^2\cdot e^{i\cdot\theta\cdot\frac{l_t}{L_t}}\cdot\left(\mathbf{OA} - e^{i\cdot\theta\cdot\frac{L}{L_t}}\cdot\mathbf{OB}\right)$$

$$\Delta_{as}(\theta_0) = 0.093 + 0.285i\,mm$$

Contribution de la courbe terminale externe

$$\Delta_{t1}(\theta) := \frac{i\cdot\theta}{L_t}\cdot r_t\cdot\int_0^{\frac{\pi}{2}} z_{0t1}(\alpha_t)\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot r_t\cdot\alpha_t\right)d\alpha_t \qquad \Delta_{t1}(\theta_0) = -0.091 + 0.217i\,mm$$

$$\Delta_{t2}(\theta) := \frac{i\cdot\theta}{L_t}\cdot\int_{-r_t}^{r_t} z_{0t2}(x)\cdot exp\left[i\cdot\frac{\theta}{L_t}\cdot\left(r_t\cdot\frac{\pi}{2} + r_t - x\right)\right]dx \qquad \Delta_{t2}(\theta_0) = -0.168 - 0.015i\,mm$$

$$\Delta_{t3}(\theta) := \frac{i\cdot\theta}{L_t}\cdot r_t\cdot\int_0^{\frac{\pi}{2}} z_{0t3}(\beta_t)\cdot exp\left[i\cdot\frac{\theta}{L_t}\cdot\left(r_t\cdot\frac{\pi}{2} + 2\cdot r_t + r_t\cdot\beta_t\right)\right]d\beta_t \qquad \Delta_{t3}(\theta_0) = -0.051 - 0.229i\,mm$$

$$\Delta_t(\theta) := \Delta_{t1}(\theta) + \Delta_{t2}(\theta) + \Delta_{t3}(\theta) \qquad \Delta_t(\theta_0) = -0.31 - 0.027i\,mm$$

Approximations

$$s_{t1}(\alpha_t) := r_t\cdot\alpha_t \qquad f_{t1}(\theta,\alpha_t) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t1}(\alpha_t)}{L_t}\right) \qquad f'_{t1}(\theta,\alpha_t) := \frac{-\theta^2}{L_t}\cdot r_t\cdot exp\left(i\cdot\theta\cdot\frac{s_{t1}(\alpha_t)}{L_t}\right)$$

$$\mathbf{Og_{11}} := \frac{r_t}{l_t}\cdot\int_0^{\frac{\pi}{2}} z_{0t1}(\alpha_t)\,d\alpha_t \qquad \mathbf{Og_{21}} := \frac{2\cdot r_t}{l_t^{\,2}}\cdot\int_0^{\frac{\pi}{2}} r_t\cdot\alpha_t\cdot z_{0t1}(\alpha_t)\,d\alpha_t$$

$$\Delta_{at1}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_{t1}(\theta,0)\cdot\mathbf{Og_{11}} + f'_{t1}(\theta,0)\cdot\frac{l_t^{\,2}}{2\cdot r_t}\cdot\mathbf{Og_{21}}\right) \qquad \Delta_{at1}(\theta_0) = -0.091 + 0.217i\,mm$$

$$s_{t2}(x) := r_t\cdot\frac{\pi}{2} + r_t - x \qquad f_{t2}(\theta,x) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t2}(x)}{L_t}\right) \qquad f'_{t2}(\theta,x) := \frac{\theta^2}{L_t}\cdot exp\left(i\cdot\theta\cdot\frac{s_{t2}(x)}{L_t}\right)$$

$$\mathbf{Og_{12}} := \frac{1}{l_t}\cdot\int_{-r_t}^{r_t} z_{0t2}(x)\,dx \qquad \mathbf{Og_{22}} := \frac{2}{l_t^{\,2}}\cdot\int_{-r_t}^{r_t} x\cdot z_{0t2}(x)\,dx$$

$$\Delta_{at2}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_{t2}(\theta,0\cdot m)\cdot\mathbf{Og_{12}} + f'_{t2}(\theta,0\cdot m)\cdot\frac{l_t^{\,2}}{2}\cdot\mathbf{Og_{22}}\right) \qquad \Delta_{at2}(\theta_0) = -0.168 - 0.015i\,mm$$

$$s_{t3}(\beta_t) := \left(r_t\cdot\frac{\pi}{2} + 2\cdot r_t + r_t\cdot\beta_t\right) \qquad f_{t3}(\theta,\beta_t) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t3}(\beta_t)}{L_t}\right) \qquad f'_{t3}(\theta,\beta_t) := \frac{-\theta^2}{L_t}\cdot r_t\cdot exp\left(i\cdot\theta\cdot\frac{s_{t3}(\beta_t)}{L_t}\right)$$

$$\mathbf{Og_{13}} := \frac{r_t}{l_t}\cdot\int_0^{\frac{\pi}{2}} z_{0t3}(\beta_t)\,d\beta_t \qquad \mathbf{Og_{23}} := \frac{2\cdot r_t}{l_t^2}\cdot\int_0^{\frac{\pi}{2}} r_t\cdot\beta_t\cdot z_{0t3}(\beta_t)\,d\beta_t$$

$$\Delta_{\mathbf{at3}}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_{t3}(\theta,0)\cdot\mathbf{Og_{13}} + f'_{t3}(\theta,0)\cdot\frac{l_t^2}{2\cdot r_t}\cdot\mathbf{Og_{23}}\right) \qquad \Delta_{\mathbf{at3}}(\theta_0) = -0.051 - 0.229i\,mm$$

$$\mathbf{Og_1} := \mathbf{Og_{11}} + \mathbf{Og_{12}} + \mathbf{Og_{13}} \qquad \mathbf{Og_1} = 1.945i\,mm \qquad \xi_{0t} = 0\,mm \qquad \eta_{0t} = 1.945\,mm$$

$$\Delta_{\mathbf{at}}(\theta) := \Delta_{\mathbf{at1}}(\theta) + \Delta_{\mathbf{at2}}(\theta) + \Delta_{\mathbf{at3}}(\theta) \qquad \Delta_{\mathbf{at}}(\theta_0) = -0.31 - 0.027i\,mm$$

Contribution de la courbe terminale interne $\quad s_{t'}(\alpha_{t'}) := r_t\cdot\alpha_{t'} + L + l_t \qquad \alpha_B = 234\,deg$

$$\Delta_{\mathbf{t'1}}(\theta) := \frac{i\cdot\theta}{L_t}\cdot\int_0^{\frac{\pi}{2}} z_{0t'1}(\alpha_{t'})\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot s_{t'}(\alpha_{t'})\right)\cdot r_t\,d\alpha_{t'} \qquad \Delta_{\mathbf{t'1}}(\theta_0) = -0.093 - 0.216i\,mm$$

$$\Delta_{\mathbf{t'2}}(\theta) := \frac{i\cdot\theta}{L_t}\cdot\int_{-r_t}^{r_t} z_{0t'2}(x')\cdot exp\left[i\cdot\frac{\theta}{L_t}\cdot\left(s_{t'}\left(\frac{\pi}{2}\right) + r_t - x'\right)\right]dx' \qquad \Delta_{\mathbf{t'2}}(\theta_0) = 0.127 - 0.111i\,mm$$

$$\Delta_{\mathbf{t'3}}(\theta) := \frac{i\cdot\theta}{L_t}\cdot\int_0^{\frac{\pi}{2}} z_{0t'3}(\beta_{t'})\cdot exp\left[i\cdot\frac{\theta}{L_t}\cdot\left(s_{t'}\left(\frac{\pi}{2}\right) + 2\cdot r_t + r_t\cdot\beta_{t'}\right)\right]\cdot r_t\,d\beta_{t'} \qquad \Delta_{\mathbf{t'3}}(\theta_0) = 0.201 + 0.122i\,mm$$

$$\Delta_{\mathbf{t'}}(\theta) := \Delta_{\mathbf{t'1}}(\theta) + \Delta_{\mathbf{t'2}}(\theta) + \Delta_{\mathbf{t'3}}(\theta) \qquad \Delta_{\mathbf{t'}}(\theta_0) = 0.235 - 0.205i\,mm$$

Approximations

$$s_{t'1}(\alpha_{t'}) := L + l_t + r_t\cdot\alpha_{t'} \qquad f_{t'1}(\theta,\alpha_{t'}) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'1}(\alpha_{t'})}{L_t}\right) \qquad f'_{t'1}(\theta,\alpha_{t'}) := \frac{-\theta^2}{L_t}\cdot r_t\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'1}(\alpha_{t'})}{L_t}\right)$$

$$\mathbf{Og'_{11}} := \frac{r_t}{l_t}\cdot\int_0^{\frac{\pi}{2}} z_{0t'1}(\alpha_{t'})\,d\alpha_{t'} \qquad \mathbf{Og'_{21}} := \frac{2\cdot r_t}{l_t^2}\cdot\int_0^{\frac{\pi}{2}} r_t\cdot\alpha_{t'}\cdot z_{0t'1}(\alpha_{t'})\,d\alpha_{t'}$$

$$\Delta_{\mathbf{at'1}}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_{t'1}(\theta,0)\cdot\mathbf{Og'_{11}} + f'_{t'1}(\theta,0)\cdot\frac{l_t^2}{2\cdot r_t}\cdot\mathbf{Og'_{21}}\right) \qquad \Delta_{\mathbf{at'1}}(\theta_0) = -0.093 - 0.216i\,mm$$

$$s_{t'2}(x') := r_t\cdot\frac{\pi}{2} + L + l_t + r_t - x' \qquad f_{t'2}(\theta,x') := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'2}(x')}{L_t}\right) \qquad f'_{t'2}(\theta,x') := \frac{\theta^2}{L_t}\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'2}(x')}{L_t}\right)$$

$$\mathbf{Og'_{12}} := \frac{1}{l_t}\cdot\int_{-r_t}^{r_t} z_{0t'2}(x')\,dx' \qquad \mathbf{Og'_{22}} := \frac{2}{l_t^2}\cdot\int_{-r_t}^{r_t} x'\cdot z_{0t'2}(x')\,dx'$$

$$\Delta_{\mathbf{at'2}}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_{t'2}(\theta,0\cdot m)\cdot\mathbf{Og'_{12}} + f'_{t'2}(\theta,0\cdot m)\cdot\frac{l_t^2}{2}\cdot\mathbf{Og'_{22}}\right) \qquad \Delta_{\mathbf{at'2}}(\theta_0) = 0.127 - 0.111i\,mm$$

$$s_{t'3}(\beta_{t'}) := s_{t'2}(-r_t) + r_t\cdot\beta_{t'} \qquad f_{t'3}(\theta,\beta_{t'}) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'3}(\beta_{t'})}{L_t}\right) \qquad f'_{t'3}(\theta,\beta_{t'}) := \frac{-\theta^2}{L_t}\cdot r_t\cdot exp\left(i\cdot\theta\cdot\frac{s_{t'3}(\beta_{t'})}{L_t}\right)$$

$$\mathbf{Og'_{13}} := \frac{r_t}{l_t}\cdot\int_0^{\frac{\pi}{2}} z_{0t'3}(\beta_{t'})\,d\beta_{t'} \qquad \mathbf{Og'_{23}} := \frac{2\cdot r_t}{l_t^2}\cdot\int_0^{\frac{\pi}{2}} r_t\cdot\beta_{t'}\cdot z_{0t'3}(\beta_{t'})\,d\beta_{t'}$$

$$\Delta\mathbf{at'3}(\theta) := \frac{1}{L_t}\cdot\left(l_t\cdot f_{t'3}(\theta,0)\cdot\mathbf{Og'_{13}} + f'_{t'3}(\theta,0)\cdot\frac{l_t^2}{2\cdot r_t}\cdot\mathbf{Og'_{23}}\right) \qquad \Delta\mathbf{at'3}(\theta_0) = 0.201 + 0.122i\,mm$$

$$\mathbf{Og'_1} := \mathbf{Og'_{11}} + \mathbf{Og'_{12}} + \mathbf{Og'_{13}} \qquad \mathbf{Og'_1} = 1.573 - 1.143i\,mm \qquad \xi_{0t'} = 1.573\,mm \qquad \eta_{0t'} = -1.143\,mm$$

$$\Delta\mathbf{at'}(\theta) := \Delta\mathbf{at'1}(\theta) + \Delta\mathbf{at'2}(\theta) + \Delta\mathbf{at'3}(\theta) \qquad \Delta\mathbf{at'}(\theta_0) = 0.235 - 0.205i\,mm$$

Contribution du spiral entier

$$\Delta\mathbf{1}(\theta) := \Delta\mathbf{t}(\theta) + \Delta\mathbf{s}(\theta) + \Delta\mathbf{t'}(\theta) \qquad \Delta\mathbf{1}(\theta_0) = 0.018 + 0.055i\,mm$$

$$u_1(\theta) := Re(\Delta\mathbf{1}(\theta)) \qquad v_1(\theta) := Im(\Delta\mathbf{1}(\theta)) \qquad u_1(\theta_0) = 0.018\,mm \qquad v_1(\theta_0) = 0.055\,mm$$

Approximation

$$\Delta\mathbf{a}(\theta) := \Delta\mathbf{at}(\theta) + \Delta\mathbf{as}(\theta) + \Delta\mathbf{at'}(\theta) \qquad \Delta\mathbf{a}(\theta_0) = 0.017 + 0.053i\,mm$$

Calcul des réactions

$$p2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\pi}^{\pi+\psi_0} x_{0s}(\alpha)^2\cdot R_0\,d\alpha + \int_0^{\frac{\pi}{2}} x_{0t1}(\alpha_t)^2\cdot r_t\,d\alpha_t + \int_{-r_t}^{r_t} x_{0t2}(x)^2\,dx + \int_0^{\frac{\pi}{2}} x_{0t3}(\beta_t)^2\cdot r_t\,d\beta_t\right)$$

$$p2_{0s} := p2_{0s} + \frac{1}{L_t}\cdot\left(\int_0^{\frac{\pi}{2}} x_{0t'1}(\alpha_{t'})^2\cdot r_t\,d\alpha_{t'} + \int_{-r_t}^{r_t} x_{0t'2}(x')^2\,dx' + \int_0^{\frac{\pi}{2}} x_{0t'3}(\beta_{t'})^2\cdot r_t\,d\beta_{t'}\right) \qquad p2_{0s} = 12.332\,mm^2$$

$$q2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\pi}^{\pi+\psi_0} y_{0s}(\alpha)^2\cdot R_0\,d\alpha + \int_0^{\frac{\pi}{2}} y_{0t1}(\alpha_t)^2\cdot r_t\,d\alpha_t + \int_{-r_t}^{r_t} y_{0t2}(x)^2\,dx + \int_0^{\frac{\pi}{2}} y_{0t3}(\beta_t)^2\cdot r_t\,d\beta_t\right)$$

$$q2_{0s} := q2_{0s} + \frac{1}{L_t}\cdot\left(\int_0^{\frac{\pi}{2}} y_{0t'1}(\alpha_{t'})^2\cdot r_t\,d\alpha_{t'} + \int_{-r_t}^{r_t} y_{0t'2}(x')^2\,dx' + \int_0^{\frac{\pi}{2}} y_{0t'3}(\beta_{t'})^2\cdot r_t\,d\beta_{t'}\right) \qquad q2_{0s} = 11.977\,mm^2$$

$$k_{0s} := \frac{1}{L_t}\cdot\int_{\pi}^{\pi+\psi_0} x_{0s}(\alpha)\cdot y_{0s}(\alpha)\cdot R_0\,d\alpha$$

$$k_{0s} := k_{0s} + \frac{1}{L_t}\cdot\left(\int_0^{\frac{\pi}{2}} x_{0t1}(\alpha_t)\cdot y_{0t1}(\alpha_t)\cdot r_t\,d\alpha_t + \int_{-r_t}^{r_t} x_{0t2}(x)\cdot y_{0t2}(x)\,dx + \int_0^{\frac{\pi}{2}} x_{0t3}(\beta_t)\cdot y_{0t3}(\beta_t)\cdot r_t\,d\beta_t\right)$$

$$k_{0s} := k_{0s} + \frac{1}{L_t}\cdot\left(\int_0^{\frac{\pi}{2}} x_{0t'1}(\alpha_{t'})\cdot y_{0t'1}(\alpha_{t'})\cdot r_t\, d\alpha_{t'} + \int_{-r_t}^{r_t} x_{0t'2}(x')\cdot y_{0t'2}(x')\, dx' + \int_0^{\frac{\pi}{2}} x_{0t'3}(\beta_{t'})\cdot y_{0t'3}(\beta_{t'})\cdot r_t\, d\beta_{t'}\right)$$

$$\mathbf{S_0} := \frac{L}{E\cdot I_{33}}\cdot\begin{pmatrix} q2_{0s} & -k_{0s} \\ -k_{0s} & p2_{0s}\end{pmatrix} \qquad \mathbf{R'}(\theta) := \mathbf{S_0}^{-1}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta)\end{pmatrix} \qquad \mathbf{R'}(\theta_0) = \begin{pmatrix} 1.7\times 10^{-5} \\ 4.822\times 10^{-5}\end{pmatrix} N$$

$$\left|\mathbf{R'}(\theta_0)\right| = 5.113\times 10^{-5}\, N$$

Approximations

$$\sigma 2 := \frac{1}{L_t}\cdot\left[\int_{\pi}^{\pi+\psi_0} \left(\left|z_{0s}(\alpha)\right|\right)^2\cdot R_0\, d\alpha + \int_0^{\frac{\pi}{2}} \left(\left|z_{0t1}(\alpha_t)\right|\right)^2\cdot r_t\, d\alpha_t + \int_{-r_t}^{r_t} \left(\left|z_{0t2}(x)\right|\right)^2 dx + \int_0^{\frac{\pi}{2}} \left(\left|z_{0t3}(\beta_t)\right|\right)^2\cdot r_t\, d\beta_t\right]$$

$$\sigma 2 := \sigma 2 + \frac{1}{L_t}\cdot\left[\int_0^{\frac{\pi}{2}} \left(\left|z_{0t'1}(\alpha_{t'})\right|\right)^2\cdot r_t\, d\alpha_{t'} + \int_{-r_t}^{r_t} \left(\left|z_{0t'2}(x')\right|\right)^2 dx' + \int_0^{\frac{\pi}{2}} \left(\left|z_{0t'3}(\beta_{t'})\right|\right)^2\cdot r_t\, d\beta_{t'}\right]$$

$$\sigma 2 = 24.309\, mm^2 \qquad R'(\theta) := \frac{E\cdot I_{33}}{L}\cdot\frac{2}{\sigma 2}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta)\end{pmatrix} \qquad R'(\theta_0) = \begin{pmatrix} 1.578\times 10^{-5} \\ 4.858\times 10^{-5}\end{pmatrix} N \qquad \left|R'(\theta_0)\right| = 5.108\times 10^{-5}\, N$$

Perturbation de période - spiral non déformé en position de repos

$$X(\theta) := \frac{\left(\left|\mathbf{\Delta_1}(\theta)\right|\right)^2}{\sigma 2} \qquad \gamma(\theta) := \frac{d}{d\theta}X(\theta) \qquad Delta(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi} \gamma(\theta_0\cdot\cos(\varphi))\cdot\cos(\varphi)\, d\varphi$$

$$\mu(\theta_0) := -86400\cdot Delta(\theta_0) \qquad \boxed{\mu(\theta_0) = 0.429} \qquad \boxed{\mu(180\cdot deg) = 0.157}$$

$$X(\theta) := \frac{\left(\left|\mathbf{\Delta_a}(\theta)\right|\right)^2}{\sigma 2} \qquad \gamma(\theta) := \frac{d}{d\theta}X(\theta) \qquad \delta_a(\theta_0) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi} \gamma(\theta_0\cdot\cos(\varphi))\cdot\cos(\varphi)\, d\varphi$$

$$\mu_a(\theta_0) := -86400\cdot\delta_a(\theta_0) \qquad \boxed{\mu_a(\theta_0) = 0.398} \qquad \boxed{\mu_a(180\cdot deg) = 0.157}$$

$$\theta_m := 180\cdot deg, 190\cdot deg .. 360\cdot deg$$